Comparison of the Resonant Frequency in Graphene and Metallic Nano-antennas

Ignacio Llatser[*,†], Christian Kremers[†], Albert Cabellos-Aparicio[*], Eduard Alarcón[*] and Dmitry N. Chigrin[†]

[*]*Nanonetworking Center in Catalunya (N3Cat), Universitat Politècnica de Catalunya, Jordi Girona 1-3, Campus Nord, D6-008 08034 Barcelona, Spain*
[†]*Theoretical Nano-Photonics Group, Institute of High-Frequency and Communication Technology, Faculty of Electrical, Information and Media Engineering, University of Wuppertal, Rainer-Gruenter-Str. 21, D-42119 Wuppertal, Germany*

Abstract. The scalability of the resonant frequency in graphene-based nano-antennas (graphennas) and metallic nano-antennas as a function of their length is analyzed. The antenna resonances are obtained by combining the dispersion relation in free-standing graphene and gold layers, respectively, with the resonance condition. Graphennas are found to have a better scalability in terms of their resonant frequency with respect to metallic nano-antennas as their length is reduced to a few μm. These results open up the possibility to build micrometer-sized graphennas that resonate in the terahertz band, at a frequency up to two orders of magnitude below metallic nano-antennas with the same size. Graphennas are thereby envisaged to enable the implementation of wireless communications among nanosystems.

Keywords: light scattering, nano-antenna, terahertz radiation, graphene
PACS: 78.67.Wj, 84.40.Ba, 73.20.Mf

Graphene, a flat monolayer of carbon atoms tightly packed in a two-dimensional honeycomb lattice, has recently attracted the attention of the research community due to its novel mechanical, thermal, chemical, electronic and optical properties [1]. Because of its unique characteristics, graphene is envisaged to enable a plethora of new potential applications, ranging from ultra high-speed transistors [2] to transparent solar cells [3].

Among these, a particularly promising application is graphene-enabled wireless communications. Owing to its ability to support Surface-Plasmon Polariton (SPP) waves [4], graphene is seen as the enabling technology for this emerging field. Indeed, a graphene-based plasmonic nano-antenna, or *graphenna*, (shown in Fig. 1) with lateral dimensions of just a few micrometers is predicted to resonate in the terahertz band [5, 6], at a dramatically lower frequency and with a higher radiation efficiency with respect to their metallic counterparts. By allowing the implementation of antennas with such a small size and a relatively low resonant frequency, graphennas are envisaged to enable wireless communications among nanosystems [7].

With the purpose of studying the novel radiation properties of graphennas, in our previous work [6] we modeled a graphenna as a graphene patch over a dielectric substrate. The graphene patch was characterized by means of its surface conductivity, calculated using the Kubo formalism [8, 9]. Within the random-phase approximation, the surface conductivity of graphene can be rep-

FIGURE 1. Schematic diagram of a graphene-based plasmonic nano-antenna, or *graphenna*.

resented in a local, Drude-like form. Once the graphene conductivity has been modeled, the relationship between the frequency of SPP waves in graphennas and their wavelength can be found by means of the dispersion relation of a graphene layer placed on the air-dielectric interface [10]:
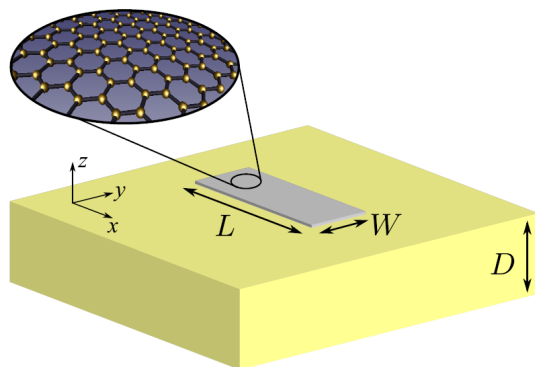
$$\frac{1}{\sqrt{\beta^2 - \frac{\omega^2}{c^2}}} + \frac{\varepsilon}{\sqrt{\beta^2 - \varepsilon\frac{\omega^2}{c^2}}} = -\mathrm{i}\frac{\sigma(\omega)}{\omega\varepsilon_0} \qquad (1)$$

where β and ω are the wavevector and angular frequency of the SPP wave, and ε is the dielectric constant of the

substrate. The resonant frequency f_0 is then obtained by combining the SPP dispersion relation with the antenna resonance condition [6]:

$$L_{\text{eff}} = L + 2\delta L = \frac{\pi}{\beta} = \frac{c}{2f_0} \qquad (2)$$

where the effective antenna length L_{eff} is obtained from the antenna length L and the penetration length of the electric field outside the antenna δL, and c stands for the speed of light.

In this context, in order to quantify the advantages of graphennas with respect to metallic nano-antennas, it is of particular interest to investigate their radiation properties. In this work, we compare the resonant frequency of both antenna types and, in particular, we analyze the scalability of their resonant frequency with respect to their length. Interestingly, we will observe that in the terahertz range not only graphennas resonate at a frequency up to two orders of magnitude lower than metallic nano-antennas, but also the resonant frequency of graphennas increases at a slower pace than that of metallic nano-antennas as their size is reduced.

In order to study the resonant frequency of a metallic nano-antennas, we will consider a patch antenna consisting of a free-standing gold slab placed on an air-dielectric interface, a structure commonly used for plasmonic nano-antennas in the optical range. The resonant frequency of this antenna can be found by analyzing its dispersion relation, given by the following expression [11]:

$$\tan\left(\Delta\sqrt{1-b}\right) = \frac{\frac{n_g^2}{n_s^2}\sqrt{\frac{b}{1-b}} + n_g^2\sqrt{\frac{b+a}{1-b}}}{1 - \frac{n_s^2}{n_g^4}\frac{\sqrt{b(b+a)}}{1-b}} \qquad (3)$$

where Δ stands for the thickness of the gold slab, n_g is the refractive index of gold, n_s that of the substrate, $a = (n_s^2 - 1)/(n_g^2 - n_s^2)$ is defined as the asymmetry measure and $b = (\beta^2/k_0^2 - n_s^2)/(n_g^2 - n_s^2)$ is the normalized mode index.

The resonant frequencies of both antennas can then be obtained by combining the respective dispersion relations (1) and (3) with the resonance condition (2). Furthermore, they can also be calculated by numerical simulation [12], measuring the frequency for which the numerically calculated absorption cross section of the antenna is maximized.

After solving the dispersion relations of graphennas and metallic nano-antennas, we find that their resonant frequencies show a very different behavior in the frequency range of interest. Figure 2 compares the resonant frequency of graphene and metallic nano-antennas as a function of their length. The antennas are modeled as simple free-standing rectangular patches without dielectric substrate; in the case of metallic (gold) antennas,
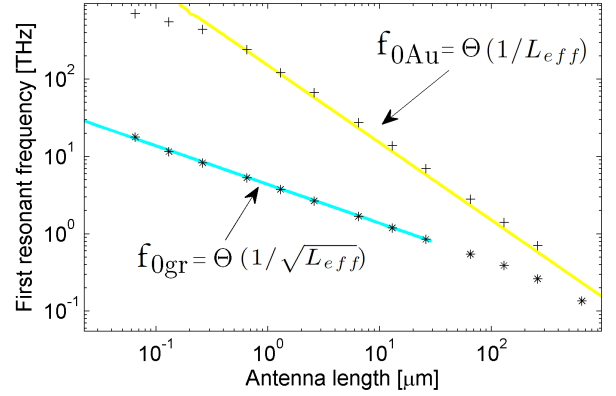

FIGURE 2. Scalability of the resonant frequency of graphene (blue line) and metallic (yellow line) nano-antennas as a function of their length. The simulation results for graphene (stars) and metallic (crosses) antennas are also shown.

we assume a thickness of $\Delta = 20$ nm, in agreement with recent experimental works [13]. The blue line shows the first resonant frequency of a graphenna, as obtained from equation (1), while the yellow line corresponds to the resonance of a metallic nano-antenna, calculated by means of its respective dispersion relation (3).

We observe that, in the terahertz range, not only the resonant frequency of a graphenna is more than one order of magnitude lower than that of a metallic nano-antenna with the same size, but also the scalability of the antenna resonant frequency as a function of its length is very different. It can be found by curve fitting that the resonant frequency of a metallic nano-antenna $f_{0\text{Au}}$ is inversely proportional to its length in the considered frequency range. In a graphenna, however, we observe a different trend; its resonant frequency $f_{0\text{gr}}$ scales inversely proportional to the square root of the antenna length:

$$f_{0\text{Au}} = \Theta\left(\frac{1}{L_{\text{eff}}}\right) \qquad f_{0\text{gr}} = \Theta\left(\frac{1}{\sqrt{L_{\text{eff}}}}\right). \qquad (4)$$

Interestingly, the scalability of the resonant frequency seen in graphennas is also different from the behavior observed in plasmonic nano-antennas made of noble metals and operating in the optical domain, whose resonant frequency is inversely proportional to the antenna length [14–16]. Indeed, their unique scalability trend turns out to be one of the strongest factors that motivate the use of graphennas to implement wireless communications among nanosystems. Indeed, as the graphenna size is reduced, its resonant frequency increases at a slower pace as compared to a metallic nano-antenna. For instance, as shown in Figure 2, metallic nano-antennas with a length of 1 µm resonate at a frequency of 150 THz, in the short-wavelength infrared range. By comparison, a

graphenna with the same size would have a resonant frequency of only 3.8 THz, almost two orders of magnitude below a metallic nano-antenna. In short, this study confirms that graphennas of micrometric size will be able to radiate electromagnetic waves in the terahertz band, at a much lower frequency and therefore with a higher transmission range than equivalent metallic nano-antennas.

Moreover, the obtained dependences for the resonances of graphene and gold antennas have been validated by numerical simulation. Graphennas are modeled as a free-standing graphene patch with an infinite width. Graphene is characterized as an equivalent surface impedance [6], with a conductivity obtained with a Drude model. On the other hand, metallic nano-antennas are modeled as a free-standing thin gold slab with an infinite width and a thickness of 20 nm. The conductivity of gold was also obtained with a Drude model.

In Figure 2, the crosses show the simulation results of the resonant frequency of graphennas with a length ranging from 50 nm to 1 mm. The penetration length δL has been estimated as a 15% of the antenna length, based on the numerical simulations. The same results for metallic nano-antennas are shown as stars. As it can be observed, in both cases the simulation results show a very good agreement with the resonant frequencies obtained by solving the respective dispersion relations. Moreover, the simulations allow measuring the evolution of the antenna resonant frequency in the regions where the numerical results diverge from the scalability trends observed by solving the dispersion relation. In particular, as it can be seen in Figure 2, the scalability of the resonant frequency of graphennas with a length larger than 100 μm is no longer $f_{0gr} = \Theta\left(1/\sqrt{L_{eff}}\right)$, but it tends to converge with the behavior observed in metallic antennas. Moreover, for metallic nano-antennas with a length smaller than 0.2 μm, as the resonant frequency reaches the threshold at which SPP waves start appearing in gold, its value starts diverging from the scalability trend $f_{0Au} = \Theta\left(1/L_{eff}\right)$ observed for lower frequencies.

To conclude, we have analyzed the scalability of the resonant frequency in graphene-based plasmonic nano-antennas (*graphennas*) and metallic nano-antennas as a function of their length, for sizes ranging from 50 nm to 1 mm. We have obtained the resonant frequency in these antennas by combining the dispersion relation in a free-standing graphene layer and a gold slab, respectively, with the antenna resonance condition. Moreover, we have validated these results by numerical simulation.

The obtained results demonstrate that micrometer-sized graphennas resonate in the terahertz band, consistently with theoretical predictions [5]. More importantly, we have found that the resonant frequency in graphennas increases more slowly than in metallic nano-antennas as the antenna length is reduced. In particular, the derived scalability trends show that, when consid-

ering submicrometer-length antennas, the resonant frequency of graphennas is up to two orders of magnitude lower than that of metallic nano-antennas with the same size.

These results open up the possibility to implementation of wireless communication among nanosystems by means of graphennas with dimensions of a few μm, thereby enabling the novel paradigm of graphene-enabled wireless communications.

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